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Ensuring Network Connectivity for Nonholonomic Robots During Decentralized Rendezvous

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Abstract— **In a multi-robot system, robots are typically required to collaborate over a communication network to achieve objectives cooperatively. Due to the limited communication and sensing capabilities on each robot, the cooperative objective must be accomplished while ensuring that specified robots stay within each other's sensing and communication ranges and that the overall network remains connected. In this paper, a dipolar navigation function and corresponding time-varying continuous controller is developed for repositioning and reorienting a group of wheeled robots with nonholonomic constraints. Only local sensing feedback information from neighbors is used to navigate the robots and maintain network connectivity, which indicates that communication is available when required for various tasks, but communication is not required for navigation. Simulation results demonstrate the performance of the developed approach.**

I. Introduction

Coordination and collaboration are crucial to the performance of multi-agent platforms in various applications. Agents are required to communicate and coordinate their movements with other agents to achieve collective tasks in either a centralized or a decentralized manner. In a centralized system, one algorithm determines and communicates the next required movement for each agent. However, the centralized approach is not practical in some applications due to the high computation load and the potential for compromised communication with or demise/corruption of the central controller. In a decentralized approach, each agent makes an independent decision based on local information from neighbors, which requires less computational effort and is more robust to the completion of desired tasks compared with centralized control. However, in decentralized control, challenges arising from performing cooperative tasks for the global network using local feedback can cause the network to partition. When the network partitions, communication between groups of agents can be permanently severed, leading to mission failure.

Results such as [1]–[9] are motivated to maintain the network connectivity in the application of formation control, flocking, consensus and other tasks. In [1], [5], [7], a potential field-based centralized approach is developed to ensure the connectivity of a group of agents which requires global knowledge of the complete network structure to determine the control for each agent, while distributed control laws using local information from neighbors are investigated to prevent the partition of the underlying graph in the work of [3], [8]–[11]. One common feature in most of the aforementioned work is that only linear models of motion are taken into account, i.e., the first order integrator. In this paper, a group of wheeled mobile robots with nonholonomic constraints are considered. Control design for the stabilization of a single robot with nonholonomic constraints has been extensively studied in the past decades [12], [13]. However, such controllers may not be applicable for a networked multi-robot system with a cooperative objective, e.g., maintaining network connectivity. In this work, assuming a range sensor (e.g., camera) provides local feedback of the relative trajectory of other robots within a limited sensing region and a transceiver is used to broadcast information to immediate neighbors on each robot, the objective is to reposition and reorient a group of wheeled robots with nonholonomic constraints to a common setpoint with a desired orientation while maintaining network connectivity during the evolution.

The navigation function, a particular class of potential functions developed in [14] and [15], is designed so that the negative gradient field does not have local minima, and the closed-loop navigation function techniques guarantee convergence to the global minimum (i.e., the control objective). The navigation function framework is widely used in the application of multi-robot systems, such as [16] and [17], to achieve cooperative objectives (e.g., formation control) under the assumption that agents with first order integrator dynamics can always communicate (i.e., the graph nodes are assumed to remain connected). When considering nonholonomic navigation to the destination with desired orientation, a dipolar navigation function was proposed and a discontinuous time-invariant controller was developed to navigate a single robot in [18]. The work in [18] was then extended to a multi-robot system with both holonomic and nonholonomic constraints in [19] and extended to navigate a nonholonomic system in three dimensions in [20]. However, only a time-invariant discontinuous controller was developed in [18]–[20]. In [6], when considering the maintenance of the network connectivity, based on the work of [18], a discontinuous controller was used to steer a multi-robot system with nonholonomic

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constraints to rendezvous at a common position. However, each robot can only achieve the destination with arbitrary orientation and has to reorient at the destination. Moreover, the multi-robot system can only converge to a destination which depends on the initial deployment in [6].

In this work, based on our previous work in [21], a decentralized continuous time-varying controller, using only local sensing feedback from its one-hop neighbors, is designed to stabilize a group of wheeled mobile robots with nonholonomic constraints at a specified common setpoint with a desired orientation. A distinguishing feature of this work is that it also considers a cooperative objective of maintaining the network connectivity during network regulation. An advantageous feature of the developed decentralized controller is that, using local sensing information, no inter-agent communication is required (i.e., communication-free global decentralized group behavior). That is, network connectivity is maintained so that the radio communication is available when required for various tasks, but communication is not required for navigation. Using the navigation function framework, the multi-robot system is guaranteed to maintain connectivity and be stabilized at a common destination with a desired orientation without being trapped by local minima from almost all initial conditions, except for a set of measure zero. Moreover, the result can be extended to other applications by replacing the objective function in the navigation function to accommodate different tasks, such as formation control, flocking, and other applications.

II. Problem Formulation

Consider a networked multi-robot system composed of N Wheeled Mobile Robots (WMRs) operating in a workspace $\mathcal{F}$, where $\mathcal{F}$ is a bounded disk area with radius R_w, and $\partial\mathcal{F}$ denotes the boundary of $\mathcal{F}$. In the workspace $\mathcal{F}$, each robot moves according to the following nonholonomic kinematics:

$$\dot{q}_i = \begin{bmatrix} \cos\theta_i & 0 \\ \sin\theta_i & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_i(t) \\ \omega_i(t) \end{bmatrix}, \; i = 1, \cdots, N \qquad (1)$$

where $q_i(t) \triangleq \begin{bmatrix} p_i^T(t) & \theta_i(t) \end{bmatrix}^T \in \mathbb{R}^3$ denotes the states of robot i, with $p_i \triangleq \begin{bmatrix} x_i(t) & y_i(t) \end{bmatrix}^T \in \mathbb{R}^2$ denoting the position of robot i, and $\theta_i \in (-\pi, \pi]$ denoting its orientation with respect to the global coordinate frame in the workspace $\mathcal{F}$. In (1), $v_i(t), \omega_i(t) \in \mathbb{R}$ are the control inputs, representing the linear and angular velocity of robot i, respectively.

Assume that each robot has a limited communication and sensing capability encoded by a disk area with radius R_c and R_s respectively, and $R_c \geq R_s$, which ensures that two robots are able to communicate with each other as long as they can sense each other. For simplicity and without loss of generality, it is assumed that the sensing area coincides with the communication area (i.e., $R_c = R_s = R$) in the following development. Two moving robots can communicate with and sense each other as long as they stay within a distance of R. Further, all the robots are assumed to have equal actuation capabilities. The interaction among the WMRs is modeled as an undirected graph $\mathcal{G}(t) = (\mathcal{V}, \mathcal{E}(t))$, where $\mathcal{V} = \{1, \cdots, N\}$ denotes the set of nodes, and $\mathcal{E}(t) = \{(i,j) \in \mathcal{V} \times \mathcal{V} | \, d_{ij} \leq R\}$ denotes a set of time-varying edges. In graph $\mathcal{G}(t)$, each node $i \in \mathcal{V}$ represents a robot located at a position p_i, and an undirected edge $(i,j) \in \mathcal{E}$ exists between node i and j in $\mathcal{G}(t)$ if their relative distance $d_{ij} \triangleq \|p_i - p_j\| \in \mathbb{R}^+$ is less than R, which indicates that node i and j are able to access the states (i.e., position and orientation) of each other through local sensing and information exchange. The neighbor set of node i is denoted as $\mathcal{N}_i = \{j \mid (i,j) \in \mathcal{E}\}$, which includes the nodes that can be sensed and reached for communication. Since the graph $\mathcal{G}(t)$ is undirected, $i \in \mathcal{N}_j \iff j \in \mathcal{N}_i$ for $\forall i$, $j \in \mathcal{V}$, $i \neq j$. Due to the limited sensing and communication capabilities, node i only knows the states of those nodes within its sensing rage and can only communicate with nodes within its communication range. Once node j moves out of the sensing and communication zone of node i, node i will no longer share information with node j directly, which may lead to mission failure. Hence, to complete desired tasks, maintaining connectivity of the underlying graph is necessary.

The main objectives in this work are to derive a set of distributed controllers using only local information (i.e., the states of the other robots within its sensing area) to lead the group of WMRs to rendezvous at a common destination p^* with a desired orientation θ_i^*, i.e., $q_i^* = \begin{bmatrix} (p^*)^T & \theta_i^* \end{bmatrix}^T \; \forall i$ in the workspace $\mathcal{F}$, while guaranteeing the underlying graph $\mathcal{G}(t)$ remains connected during the system evolution, provided the given initial graph is connected. To achieve these goals, the following assumptions are required in the subsequent development.

Assumption 1: The initial graph $\mathcal{G}$ is connected, and those initial conditions do not coincide with unstable equilibria (i.e., saddle points).

Assumption 2: The destination p^* and desired orientation θ_i^* is known for each robot and achievable, which indicates that the destination will not meet any constraints, i.e., coincide with the workspace boundary, or lead to the partition of the underlying graph.

III. Control Design

A. Dipolar Navigation Function

Artificial potential field-based methods that use attractive and repulsive potentials have been widely used to control multi-robot systems. Due to the existence of local minima when attractive and repulsive force are combined, robots can be trapped by local minima and are not guaranteed to reach the global minimum of the potential field. A navigation function is a particular category of potential functions where the potential field does not have local minima and the negative gradient vector field of the potential field guarantees almost global convergence to a desired destination, along with (guaranteed) collision avoidance. In [14] and [15], a navigation function φ is defined as a map from the workspace $\mathcal{F}$ to an interval $[0, 1]$, which is 1) smooth on $\mathcal{F}$ (at least a $\mathcal{C}^2$ function), 2) admissible on $\mathcal{F}$ (uniformly maximal on $\partial\mathcal{F}$ and constraint boundary), 3) polar on $\mathcal{F}$ (q_d is a unique minimum), and 4) a Morse function (critical points of the navigation function are non-degenerate). Property 2) establishes that the generated trajectories are collision-free, since the resulting vector field is maximum on the constraint and workspace boundary. Property 3) implies that all initial conditions are either brought to a saddle point or to the unique minimum q_d, while property 4) ensures that the initial conditions that bring the system to saddle points are sets of measure zero, and all initial conditions away from sets of measure zero are brought to the unique minimum.

The navigation function introduced in [14] and [15] ensures global convergence of the closed-loop system; however, the approach is not suitable for nonholonomic systems, since the feedback law generated from the gradient of the navigation function can lead to undesired behavior. To overcome the undesired behaviors, the original navigation function was extended to a Dipolar Navigation Function in [18] and [22], where the flow lines created in the potential field resemble a dipole, so that the flow lines are all tangent to the desired orientation at the origin and the vehicle can achieve the desired orientation.

To maintain network connectivity and navigate the robots to a common destination with a desired orientation using local information, the dipolar navigation function in [18] and [22] is modified for each node i as $\varphi_i : \mathcal{F} \to [0,1]$,

$$\varphi_i = \frac{\gamma_i}{\left(\gamma_i^{\alpha} + H_i \cdot \beta_i\right)^{1/\alpha}}, \tag{2}$$

where $\alpha \in \mathbb{R}^{+}$ is a tuning parameter. The goal function $\gamma_i(t) : \mathbb{R}^2 \to \mathbb{R}^{+}$ in (2) encodes the control objective of achieving the desired destination for node i, specified by the distance from $p_i(t) \in \mathbb{R}^2$ to the common destination $p^* \in \mathbb{R}^2$, which is designed as

$$\gamma_i = \left\| p_i(t) - p^* \right\|^2. \tag{3}$$

The factor $H_i(t) \in \mathbb{R}^{+}$ in (2) creates the repulsive potential of an artificial obstacle to align the trajectories at the destination with the desired orientation. The repulsive potential factor is designed as

$$H_i = \varepsilon_{nh} + \prod\nolimits_{j \in \mathcal{N}_i} \eta_j, \tag{4}$$

where ε_{nh} is a small positive constant, and $\eta_i(t) \in \mathbb{R}^{+}$ is designed as

$$\eta_i = \left((p_i - p^*)^T \cdot n_{di} \right)^2, \tag{5}$$

where $n_{di} = \begin{bmatrix} \cos(\theta_i^*) & \sin(\theta_i^*) \end{bmatrix}^T \in \mathbb{R}^2$. To ensure connectivity of the existing links between two nodes and restrict the motion of each node in the specified workspace, the constraint function $\beta_i : \mathbb{R}^{2N} \to [0,1]$ in (2) is designed as

$$\beta_i = B_{i0} \cdot \prod\nolimits_{j \in \mathcal{N}_i} b_{ij}. \tag{6}$$

Specifically, the constraint function in (6) is designed to vanish whenever node i meets one of the constraints in the workspace, (i.e., if node i touches the workspace boundary, or departs from its neighbor nodes $j \in \mathcal{N}_i$ to a distance of R). A small disk area with radius $\delta_1 < R$ centered at node i is denoted as a collision region. To prevent a potential collision between node i and the workspace boundary $\partial\mathcal{F}$, the function $B_{i0} : \mathbb{R}^2 \to [0,1]$ in (6) is designed as

$$B_{i0} = \begin{cases} -\frac{1}{\delta_1^2} d_{i0}^2 + \frac{2}{\delta_1} d_{i0}, & d_{i0} < \delta_1 \\ 1, & d_{i0} \geq \delta_1, \end{cases} \tag{7}$$

where $d_{i0} \triangleq R_w - \|p_i\| \in \mathbb{R}$ is the relative distance of node i to the workspace boundary. To ensure connectivity of the underlying graph, an escape region for each node is defined as the outer ring of the sensing and communication area with radius r, $R - \delta_2 < r < R$, where $\delta_2 \in \mathbb{R}^{+}$ is a predetermined buffer distance. Each edge formed by node i and the adjacent node $j \in \mathcal{N}_i$ in the escape region have the potential to lose connectivity. In (6), $b_{ij} \triangleq b(p_i, p_j) : \mathbb{R}^2 \to [0,1]$ ensures connectivity of the network graph (i.e., guarantees that nodes $j \in \mathcal{N}_i$ will never leave the sensing and communication zone of node i if node j is initially connected to node i) and is designed as

$$b_{ij} = \begin{cases} 1 & d_{ij} \leq R - \delta_2 \\ -\frac{1}{\delta_2^2}(d_{ij} + 2\delta_2 - R)^2 & \\ +\frac{2}{\delta_2}(d_{ij} + 2\delta_2 - R) & R - \delta_2 < d_{ij} < R \\ 0 & d_{ij} \geq R. \end{cases} \tag{8}$$

Since γ_i and β_i are guaranteed to not be zero simultaneously by Assumption 2, the navigation function candidate in (2) achieves its minimum of 0 when $\gamma_i = 0$ and achieves its maximum of 1 when $\beta_i = 0$. Our previous work in [10] proves that the original navigation function modified to ensure connectivity, as designed in (8), is still a qualified navigation function. It is also shown in [19] that the navigation properties are not affected by the modification to a dipolar navigation with the design of (4), as long as the workspace is bounded, η_i in (5) can be bounded in the workspace, and ε_{nh} is a small positive constant. As a result, the decentralized navigation function φ_i proposed in (2) can be proven to be a qualified navigation function. See [10] and [19] for further details. From the properties of the navigation function, it is known that, if φ_i is a qualified navigation function, almost all initial positions (except for a set of measure zero points) asymptotically approach the desired destination.

B. Control Development

The desired orientation for robot i, denoted by $\theta_{di}(t)$, is defined as a function of the negative gradient of the decentralized navigation function in (2) as,

$$\theta_{di} \triangleq \arctan 2\left(-\frac{\partial \varphi_i}{\partial y_i}, \ -\frac{\partial \varphi_i}{\partial x_i} \right), \tag{9}$$

where $\arctan 2(\cdot) : \mathbb{R}^2 \to \mathbb{R}$ denotes the four quadrant inverse tangent function, and $\theta_{di}(t)$ is confined to the region of $(-\pi, \pi]$. By defining $\theta_{di}|_{p^*} = \arctan 2(0,0) = \theta_i|_{p^*}$, θ_{di}

remains continuous along any approaching direction to the goal position. Based on the definition of θ_{di} in (9)

$$\nabla_i \varphi_i = -\left\| \nabla_i \varphi_i \right\| \left[\begin{array}{cc} \cos(\theta_{di}) & \sin(\theta_{di}) \end{array} \right]^T, \tag{10}$$

where $\nabla_i \varphi_i = \left[\begin{array}{cc} \frac{\partial \varphi_i}{\partial x_i} & \frac{\partial \varphi_i}{\partial y_i} \end{array} \right]^T$ denotes the partial derivative of φ_i with respect to p_i, and $\|\nabla_i \varphi_i\|$ denotes the Euclidean norm of $\nabla_i \varphi_i$. The difference between the current orientation and the desired orientation for robot i at each time instant is defined as

$$\tilde{\theta}_i(t) = \theta_i(t) - \theta_{di}(t), \tag{11}$$

where $\theta_{di}(t)$ is generated from the decentralized navigation function in (2) and (9). Since φ_i in (2) is a qualified navigation function, the properties of a navigation function guarantees that $q_{di}(t) \to q_i^*$ as $t \to \infty$. Hence, to achieve the navigation control objective, $\theta_i(t)$ must track the desired orientation $\theta_{di}(t)$.

Based on the open-loop system in (1) and the subsequent stability analysis, the controller for each robot (i.e., the linear and angular velocity of robot i) is designed as

$$\begin{aligned} v_i &= k_v \|\nabla_i \varphi_i\| \cos\tilde{\theta}_i, & (12)\\ \omega_i &= -k_w \tilde{\theta}_i + \dot{\theta}_{di}, & (13) \end{aligned}$$

where $k_v, k_w \in \mathbb{R}^+$. The terms $\nabla_i \varphi_i$ and $\dot{\theta}_{di}$ in (12) and (13) are determined from (2) and (9) as

$$\nabla_i \varphi_i = \frac{\alpha (H_i \cdot \beta_i) \nabla_i \gamma_i - \gamma_i \nabla_i (H_i \cdot \beta_i)}{\alpha (\gamma_i^\alpha + H_i \cdot \beta_i)^{\frac{1}{\alpha}+1}}, \tag{14}$$

where $\nabla_i \gamma_i$ and $\nabla_i (H_i \cdot \beta_i)$ are bounded in the workspace $\mathcal{F}$ from (3) and (6), and

$$\dot{\theta}_{di} = k_v \cos(\tilde{\theta}_i) \left[\begin{array}{c} \sin(\theta_{di}) \\ -\cos(\theta_{di}) \end{array} \right]^T \nabla_i^2 \varphi_i \left[\begin{array}{c} \cos(\theta_i) \\ \sin(\theta_i) \end{array} \right], \tag{15}$$

where $\nabla_i^2 \varphi_i$ denotes the Hessian matrix of φ_i with respect to p_i. Substituting (12) into (1), the closed-loop system for robot i can be obtained as

$$\dot{p}_i = \left[\begin{array}{c} \dot{x}_i \\ \dot{y}_i \end{array} \right] = k_v \|\nabla_i \varphi_i\| \cos\tilde{\theta}_i \left[\begin{array}{c} \cos\theta_i \\ \sin\theta_i \end{array} \right]. \tag{16}$$

After using the fact that

$$\left[\begin{array}{cc} \cos\theta_i & \sin\theta_i \end{array} \right] \nabla_i \varphi_i = -\|\nabla_i \varphi_i\| \cos\tilde{\theta}_i, \tag{17}$$

from (10), the closed-loop error systems in (17) can be expressed as

$$\dot{p}_i = -k_v \nabla_i \varphi_i. \tag{18}$$

IV. Connectivity and Convergence Analysis

A. Connectivity Analysis

Theorem 1: Given an initially connected graph $\mathcal{G}$ composed of nodes with kinematics given by (1), the controller in (12) and (13) ensure the graph remains connected.

Proof: Consider node i located at a point $p_0 \in \mathcal{F}$ that causes $\prod_{j \in \mathcal{N}_i} b_{ij} = 0$, which will be true when either only one node j is about to disconnect from node i or when multiple nodes are about to disconnect with node i simultaneously. From (6), $\beta_i = 0$, which indicates that the navigation function designed in (2) achieves its maximum value. From the negative gradient of φ_i in (18), no open set of initial conditions can be attracted to the maxima of the navigation function [15]. Therefore, the existing edge between node i and node $j \in \mathcal{N}_i$ will be maintained for all time. ■

B. Convergence Analysis

Theorem 2: Given an initially connected graph $\mathcal{G}$ composed of nodes with kinematics given by (1), the controller in (12) and (13) ensure the robots converges to a common point with a desired orientation, in the sense that $\|p_i(t) - p^*\| \to 0$ and $\left|\tilde{\theta}_i(t)\right| \to 0$ as $t \to \infty$ $\forall i \in \mathcal{N}$, provided that the tuning parameter α in (2) is sufficient large, $\alpha > \Theta$.

Proof: Consider a Lyapunov function candidate $V(\mathbf{P}(t)) = \sum_{i=1}^N \varphi_i$, where $\mathbf{P}(t)$ is the stacked position states of all nodes, $\mathbf{P}(t) = \left[\begin{array}{ccc} p_1^T(t) & \cdots & p_N^T(t) \end{array} \right]^T$, and φ_i is the associated navigation function for node i designed in (2). The time derivative of V is

$$\begin{aligned} \dot{V} &= \sum_{i=1}^N (\nabla_i \varphi_1)^T \dot{p}_i + \cdots + \sum_{i=1}^N (\nabla_i \varphi_N)^T \dot{p}_i \\ &= \sum_{i=1}^N \sum_{j=1}^N \dot{p}_i^T (\nabla_i \varphi_j), \end{aligned}$$

which can be further separated as

$$\begin{aligned} \dot{V} &= \sum_{i:\nabla_i \varphi_i = 0} \left(\dot{p}_i^T (\nabla_i \varphi_i) + \sum_{j \neq i} \dot{p}_i^T (\nabla_i \varphi_j) \right) \quad (19) \\ &\quad + \sum_{i:\nabla_i \varphi_i \neq 0} \left(\dot{p}_i^T (\nabla_i \varphi_i) + \sum_{j \neq i} \dot{p}_i^T (\nabla_i \varphi_j) \right), \end{aligned}$$

where $\nabla_i \varphi_j, \nabla_i \varphi_i \in \mathbb{R}^2$ denote the partial derivative of φ_j and φ_i with respect to p_i, respectively.

To show the objective of $\|p_i - p^*\| \to 0$ $\forall i \in \mathcal{N}$, the set of critical points, $S = \{p_i \mid \nabla_i \varphi_i = 0 \text{ for } \forall i \in \mathcal{N}\}$ must be shown to be the largest invariant set of the stacked closed-loop system of (18). When all nodes are located at the critical points (i.e., the position of node i satisfying $\nabla_i \varphi_i = 0$) in (19), $\dot{V} = 0$, since $\dot{p}_i = 0$ from (16). For node i not located at the critical points (i.e., $\nabla_i \varphi_i \neq 0$), (19) can be rewritten as

$$\dot{V} = \sum_{i:\nabla_i \varphi_i \neq 0} \left(\dot{p}_i^T (\nabla_i \varphi_i) + \sum_{j \neq i} \dot{p}_i^T (\nabla_i \varphi_j) \right). \tag{20}$$

To show that the set of critical points is the largest invariant set, $\dot{V}$ must be strictly negative whenever there exists at least one node i such that $\nabla_i \varphi_i \neq 0$, for which it is sufficient to show that

$$\dot{p}_i^T (\nabla_i \varphi_i) + \sum_{j \neq i} \dot{p}_i^T (\nabla_i \varphi_j) < 0. \tag{21}$$

Substituting (14) and (18) into (21),yields

$$k_v (\nabla_i \varphi_i)^T (\nabla_i \varphi_i) + k_v \sum_{j \neq i} (\nabla_i \varphi_i)^T (\nabla_i \varphi_j) > 0,$$

which can be simplified as

$$\frac{1}{\alpha^2} c_1 + \frac{1}{\alpha} c_2 + c_3 > 0, \tag{22}$$

where

$$c_1 = k_v \gamma_i \nabla_i^T (H_i \beta_i) \sum\nolimits_{j \neq i} \gamma_j \nabla_i (H_j \beta_j)),$$
$$c_2 = -k_v H_i \beta_i \nabla_i^T \gamma_i \sum\nolimits_{j \neq i} \gamma_j \nabla_i (H_j \beta_j)),$$
$$c_3 = k_v (\nabla_i \varphi_i)^T (\nabla_i \varphi_i),$$

by using the fact that $\nabla_i \gamma_j = 0$ from (3) and H_i, β_i, γ_i, α are positive from (3), (4) and (6). A sufficient condition for the inequality in (22) to be satisfied is

$$-\frac{1}{\alpha^2} |c_1| - \frac{1}{\alpha} |c_2| > -c_3.$$

Hence, if $\alpha > \Theta$, where $\Theta = \max \left\{ \sqrt{\frac{|c_1|}{c_3}}, \frac{|c_2|}{c_3} \right\}$, the system converges to the set of critical points. Applying LaSalle's invariance principle, the trajectories of the system converge to the largest invariant set contained in the set

$$S = \{ \|\nabla_i \varphi_i\| = 0, \ \forall i \in \mathcal{V} \}. \tag{23}$$

The set in (20) is formed whenever the potential functions either reach the destination or a saddle point. Since φ_i in (2) is a navigation function, the saddle points of φ_i are isolated in [10]. Thus, the set of initial conditions that lead to saddle points are sets of measure zero [23]. The largest invariant set constrained is the set of destination [24]. Hence, $\|\nabla_i \varphi_i\| = 0$ indicates that $\|p_i - p^*\| \to 0$ for $\forall i$.

To show that $\left|\tilde{\theta}_i\right| \to 0$, we take the time derivative of $\tilde{\theta}_i(t)$ in (11) and use (1) to develop the open-loop orientation tracking error system as $\dot{\tilde{\theta}}_i = \omega_i - \dot{\theta}_{di}$. Using (13), the closed-loop orientation tracking error is

$$\dot{\tilde{\theta}}_i = -k_w \tilde{\theta}_i, \tag{24}$$

which has the exponentially decaying solution $\tilde{\theta}_i(t) = \tilde{\theta}_i(0) e^{-k_w t}$.

Based on (3) and (8), it is clear that $\frac{\partial \varphi}{\partial x_i}, \frac{\partial \varphi}{\partial y_i} \in \mathcal{L}_\infty$ on $\mathcal{F}$; hence, (12) can be used to conclude that $v_i(t) \in \mathcal{L}_\infty$. Provided $\dot{\theta}_{di}(t) \in \mathcal{L}_\infty$ in (15) on $\mathcal{F}$, (13) can be used to show that $\omega_i(t) \in \mathcal{L}_\infty$. ■

V. Simulation

A preliminary numerical simulation is performed in this section to demonstrate the performance of the controller developed in (12) and (13) in a scenario where a group of four mobile robots with the kinematics in (1) are navigated to the common destination $\left[(p^*)^T, \theta^* \right]^T = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$. The four mobile robots are deployed in a workspace of $R_w = 5\ m$ with an initially connected condition of $q_1^T(0) = \begin{bmatrix} -2 & 1.5 & -1.131 \end{bmatrix}$, $q_2^T(0) = \begin{bmatrix} -2.25 & 0.7 & -1.7279 \end{bmatrix}$, $q_3^T(0) = \begin{bmatrix} -2.5 & -0.7 & 1.8850 \end{bmatrix}$ and $q_4^T(0) = \begin{bmatrix} -2.25 & -1.5 & 0.9425 \end{bmatrix}$. The limited communication and sensing zone for each robot is assumed as $R = 2\ m$ and $\delta_1 = \delta_2 = 0.5\ m$. The tuning parameter α in (2) is selected as $\alpha = 1.5$, and the control gains k_v and k_w are adjusted to $k_v = 1.1$ and $k_w = 0.9$. The control law in (12) and (13) yields the simulation results shown in Fig. 1-4. Fig. 1 shows the trajectory evolution for each robot, where the robots are represented by dots, and the associated arrows indicate the current orientation. The linear and angular velocity control inputs for each robot are shown in Fig. 2. In Fig. 3, the plot of position and orientation error for each mobile robot indicates that each robot achieves the common destination with the desired orientation. The evolution of inter-robot distance is shown in Fig. 4, which implies that the connectivity of the underlying graph is maintained, since the inter-robot distance is less than the radius $R = 2\ m$ during the motion.

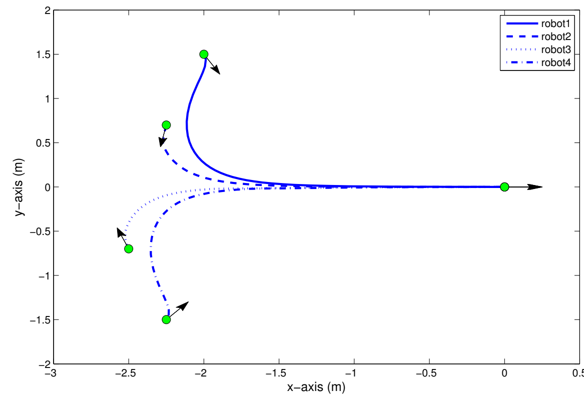

Fig. 1. The trajectory for each mobile robot.

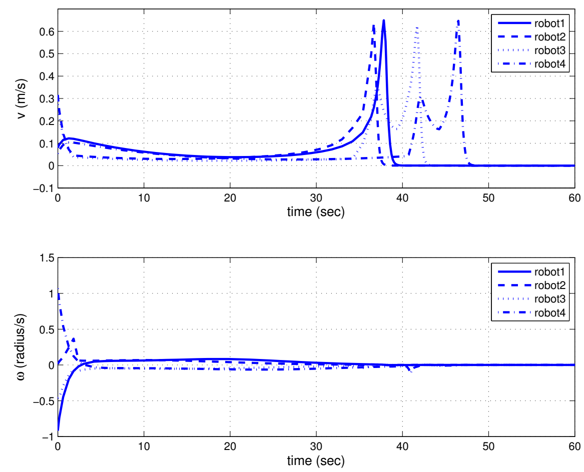

Fig. 2. Plot of linear velocity and angular velocity for each mobile robot.

VI. Conclusion

Based on the dipolar navigation function formalism, a decentralized time-varying continuous controller is developed to achieve network cooperative goals, that are navigating mobile robots to a common destination with a desired orientation and ensuring the network connectivity for all time, by using only local sensing information from one-hop neighbors. A

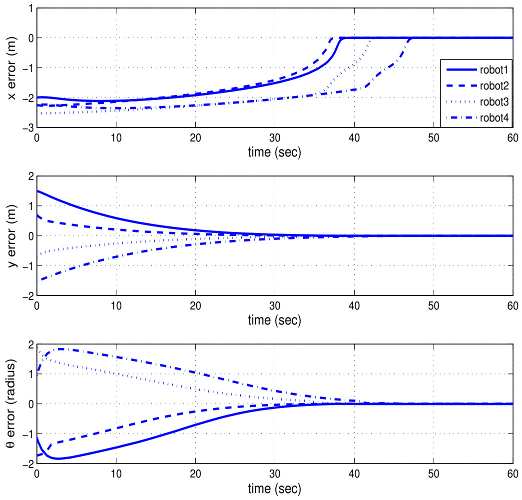

Fig. 3. Plot of position and orientation error for each mobile robot.

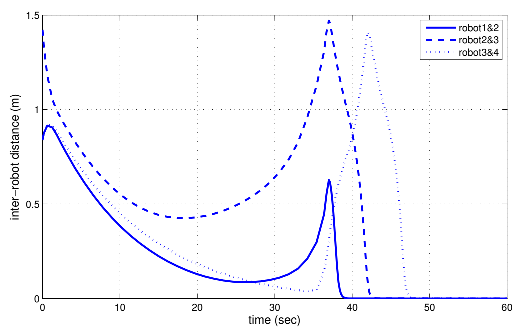

Fig. 4. The evolution of inter-robot distance.

distinguishing feature of the developed decentralized approach is that no inter-agent communication is required to complete the network rendezvous objective, which results in radio silence during the network regulation. Future efforts are focused on enabling collision avoidance with obstacles in a dynamic environment using local sensing information.

References

[1] M. Zavlanos and G. Pappas, "Potential fields for maintaining connectivity of mobile networks," *IEEE Trans. Robot.*, vol. 23, no. 4, pp. 812–816, Aug. 2007.

[2] ——, "Distributed connectivity control of mobile networks," *IEEE Trans. Robot.*, vol. 24, no. 6, pp. 1416–1428, Dec. 2008.

[3] M. Zavlanos, H. Tanner, A. Jadbabaie, and G. Pappas, "Hybrid control for connectivity preserving flocking," *IEEE Trans. Autom. Control*, vol. 54, no. 12, pp. 2869 –2875, 2009.

[4] D. Dimarogonas and K. Kyriakopoulos, "Connectedness preserving distributed swarm aggregation for multiple kinematic robots," *IEEE Trans. Robot*, vol. 24, no. 5, pp. 1213 –1223, 2008.

[5] A. Ghaffarkhah and Y. Mostofi, "Communication-aware target tracking using navigation functions - centralized case," in *Int. Conf. Robot Commun. Co-ord.*, March 31 - April 2 2009, pp. 1 –8.

[6] D. V. Dimarogonas and K. J. Kyriakopoulos, "On the rendezvous problem for multiple nonholonomic agents," *IEEE Trans. Autom. Control*, vol. 52, no. 5, pp. 916–922, May 2007.

[7] R. Olfati-Saber and R. Murray, "Consensus problems in networks of agents with switching topology and time-delays," *IEEE Trans. Autom. Control*, vol. 49, no. 9, pp. 1520–1533, Sept. 2004.

[8] D. Dimarogonas and K. Johansson, "Bounded control of network connectivity in multi-agent systems," *Control Theory Applications*, vol. 4, no. 8, pp. 1330 –1338, Aug. 2010.

[9] M. Ji and M. Egerstedt, "Distributed coordination control of multiagent systems while preserving connectedness," *IEEE Trans. Robot.*, vol. 23, no. 4, pp. 693–703, 2007.

[10] Z. Kan, A. Dani, J. M. Shea, and W. E. Dixon, "Ensuring network connectivity during formation control using a decentralized navigation function," in *Proc. IEEE Mil. Commun. Conf.*, San Jose, CA, 2010, pp. 954–959.

[11] H. G. Tanner, A. Jadbabaie, and G. J. Pappas, "Flocking in fixed and switching networks," *IEEE Trans. Autom. Control*, vol. 52, no. 5, pp. 863–868, May 2007.

[12] I. Kolmanovsky and N. McClamroch, "Developments in nonholonomic control problems," *IEEE Control Syst. Mag.*, vol. 15, no. 6, pp. 20–36, 1995.

[13] Z. Li and J. Canny, *Nonholonomic motion planning*. Kluwer Academic Pub, 1993, vol. 192.

[14] D. E. Koditschek and E. Rimon, "Robot navigation functions on manifolds with boundary," *Adv. Appl. Math.*, vol. 11, pp. 412–442, Dec 1990.

[15] E. Rimon and D. Koditschek, "Exact robot navigation using artificial potential functions," *IEEE Trans. Robot. Autom.*, vol. 8, no. 5, pp. 501–518, Oct 1992.

[16] H. Tanner and A. Kumar, "Towards decentralization of multi-robot navigation functions," in *Proc. IEEE Int. Conf. Robot. Autom.*, April 2005, pp. 4132 – 4137.

[17] M. De Gennaro and A. Jadbabaie, "Formation control for a cooperative multi-agent system using decentralized navigation functions," in *Proc. Am. Control Conf.*, June 2006, pp. 1346–1351.

[18] H. Tanner and K. Kyriakopoulos, "Nonholonomic motion planning for mobile manipulators," in *IEEE International Conference onRobotics and Automation*, vol. 2, 2000, pp. 1233 –1238 vol.2.

[19] S. Loizou and K. Kyriakopoulos, "Navigation of multiple kinematically constrained robots," *IEEE Trans. Robot*, vol. 24, no. 1, pp. 221 –231, 2008.

[20] G. Roussos, D. Dimarogonas, and K. Kyriakopoulos, "3D navigation and collision avoidance for a non-holonomic vehicle," in *Proc. Am. Control Conf.*, 2008, pp. 3512 –3517.

[21] Z. Kan, A. Dani, J. Shea, and W. E. Dixon, "Ensuring network connectivity for nonholonomic robots during rendezvous," in *Proc. IEEE Conf. Decis. Control*, Orlando, FL, 2011, pp. 2369–2374.

[22] H. Tanner, S. Loizou, and K. Kyriakopoulos, "Nonholonomic navigation and control of cooperating mobile manipulators," *IEEE Trans. Robot. Autom.*, vol. 19, no. 1, pp. 53–64, Feb 2003.

[23] J. Milnor, *Morse theory*. Princeton Univ Pr, 1963.

[24] D. Dimarogonas and K. Kyriakopoulos, "A feedback control scheme for multiple independent dynamic non-point agents," *Int. J. Control*, vol. 79, no. 12, pp. 1613–1623, 2006.